Differential equation

Solve the following differential equation:

$$xdy + ydx = \sin(x)dx$$

Solution

It is an exact differential equation:

$$xdy + (y - \sin(x))dx = 0$$

We check the symmetry condition, $Q = x$ and $P = y - \sin(x)$. Therefore $P'_y = Q'_x = 1$. I propose a function $U(x, y)$ such that $dU = U'_x dx + U'_y dy$. $U'_x = P$ and $U'_y = Q$. Integrating:

$$\int du = \int xdy = xy + J$$

$$\int du = \int y - \sin(x)dx = xy + \cos(x) + K$$

Therefore, the solution is:

$$xy + \cos(x) = C$$

$$y = \frac{C}{x} - \frac{\cos(x)}{x}$$